



LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2024

UMT 1502 – CALCULUS



Date: 13-11-2024

Dept. No.

Max. : 100 Marks

Time: 09:00 am-12:00 pm

SECTION A - K1 (CO1)

Answer ALL the Questions -

(10x1=10)

1. Answer the following

- If $y = e^{ax}$, find the n^{th} derivative.
- Write the formula to find the angle of intersection between two curves in cartesian form.
- State any two properties of definite integral.
- If $x = r \cos \theta$ and $y = r \sin \theta$, find $\frac{\partial(x, y)}{\partial(r, \theta)}$.
- Define beta function.

2. Fill in the blanks

- If $f(x, y) = y^2 + 2yx^2 + 4x - 3$, then $\frac{\partial f}{\partial x} =$ _____.
- The formula to find the radius of curvature in cartesian form is _____.
- If f is an even function, then $\int_{-a}^{+a} f(x) dx =$ _____.
- $\int_0^1 \int_1^2 xy dy dx =$ _____.
- The value of $\Gamma\left(\frac{3}{2}\right) =$ _____.

SECTION A - K2 (CO1)

Answer ALL the Questions

(10x1=10)

3. Choose the correct answer

- Let $f: R \rightarrow R$ be a function such that $f'(x) = 0$ and $f''(x) > 0$. Then at the point x , the function f is
i) increasing ii) decreasing iii) attains a maximum value iv) attains a minimum value.
- The formula to the length of subnormal is
i) $\frac{y}{\frac{dy}{dx}}$ ii) $y \frac{dy}{dx}$ iii) $y^2 \frac{dy}{dx}$ iv) None of above
- $\int_0^\pi \cos x dx =$ _____
i) 1 ii) 0 iii) -1 iv) None of above
- $\int_a^b \int_c^d f(x, y) dx dy =$ _____
i) $\int_a^b \int_c^d f(x, y) dy dx$ ii) $\int_c^d \int_a^b f(x, y) dy dx$ iii) $\int_c^d \int_a^b f(x, y) dx dy$ iv) None of above
- $\beta\left(\frac{1}{2}, \frac{1}{2}\right) =$ _____

	i) $\sqrt{\pi}$	ii) π	iii) $-\sqrt{\pi}$	iv) $-\pi$
4.	True or False			
a)	n^{th} derivative of $y = \sin x$ is $\sin\left(\frac{n\pi}{2} + x\right)$.			
b)	The slope of the curve $r = a(1 - \cos \theta)$ at $\theta = \frac{\pi}{2}$ is $\frac{a}{2}$.			
c)	$\int_0^1 \int_0^1 \int_0^1 dx dy dz = \frac{1}{3}$.			
d)	If u, v are functions of x, y then $\frac{\partial(u, v)}{\partial(x, y)} \cdot \frac{\partial(x, y)}{\partial(u, v)} = 1$.			
e)	$\Gamma(n) = n \Gamma(n-1)$.			

SECTION B - K3 (CO2)

Answer any TWO of the following

(2x10=20)

5.	Find y_n , where $y = \frac{3}{(x+1)(2x-1)}$.
6.	Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point $(1, 1)$.
7.	Evaluate $\int \frac{3x+1}{(x-1)^2(x+2)} dx$.
8.	Evaluate $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx$.

SECTION C – K4 (CO3)

Answer any TWO of the following

(2x10 = 20)

9.	If $y = \sin(m \sin^{-1} x)$, prove that $(1-x^2)y_{n+2} - (2n+1)x y_{n+1} + (m^2 - n^2)y_n = 0$.
10.	Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at the point $(2, 1)$.
11.	Prove that $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx = \frac{\pi}{4}$.
12.	a) Prove that $\beta(m, n) = \beta(n, m)$. b) Evaluate $\int_0^1 x^7 (1-x)^8 dx$. (5+5)

SECTION D – K5 (CO4)

Answer any ONE of the following

(1 x 20 = 20)

13.	a) Investigate the maximum and minimum values of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. b) Find the angle of intersection of the cardioids $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$. (10+10)
14.	a) By changing the order of integration evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx dy$. b) Evaluate $\int_0^{\frac{\pi}{2}} \log \sin x dx$. (10+10)

SECTION E – K6 (CO5)

Answer any ONE of the following

(1 x 20 = 20)

15.	a) If $u = a^3 x^2 + b^3 y^2 + c^3 z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find the minimum value of u . b) Show that the evolute of the cycloid $x = a(\theta - \sin \theta)$; $y = a(1 - \cos \theta)$ is another cycloid. (10+10)
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16.	a) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$. b) Find the relation between beta and gamma function. (10+10)
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